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CyberMath: A Shared Virtual Environment for Mathematics Exploration

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ABSTRACT

We present CyberMath, an avatar-based shared virtual environment aimed at improving the current problematic situation in mathematics education. CyberMath allows mathematics to be presented in a new and exciting way and is suitable for exploring and teaching mathematics in situations both where teacher and students are co-present and physically separated. CyberMath also supports a variety of teaching styles, ranging from traditional one-to-many lectures to teacher-supported interaction and individual off-line exploration. We also summarize the results of two initial usability studies of our system.

INTRODUCTION

The current state of mathematics education is problematic in many countries. Students have considerable difficulty in finding mathematics relevant. They also have trouble understanding abstract concepts and bridging related mathematical areas [2]. Especially worrying is the fact that students seem to perform worse in later grades than in earlier grades [12].

We are investigating whether it is possible to improve this situation through the use of new computer technology. We believe that this technology must have at least the following features:

- It must be flexible enough to allow teaching of both elementary, intermediate and advanced mathematics and geometry.
- It must allow the sharing of resources. At a given school, some subjects in mathematics may be well known only to some teachers. We would like to make it possible for these teachers to offer their expertise to several groups of students from a remote location.
- It must allow the teacher to teach in a direct manner using the technology. We believe that it is vital that a teacher is present to guide and help students. Our technology must support teacher guidance even when the teacher is situated in a separate physical location.
- It must allow students to work together in groups.
- It must allow teachers to present material that is thought to be difficult or that is hard to visualize using standard teaching tools.
- It must have a ”wow” factor. It is important that the technology is interesting to the students. It is also important that the technology facilitates a continuing interest and encourages exploration. This means that it has to be flexible, extendable and easily reconfigurable to suit the needs of both teachers and students.
- It should take advantage of computer hardware that exists in the schools today.

This paper introduces CyberMath, a shared virtual environment that runs on standard desktop PCs and aims to implement all of these features.

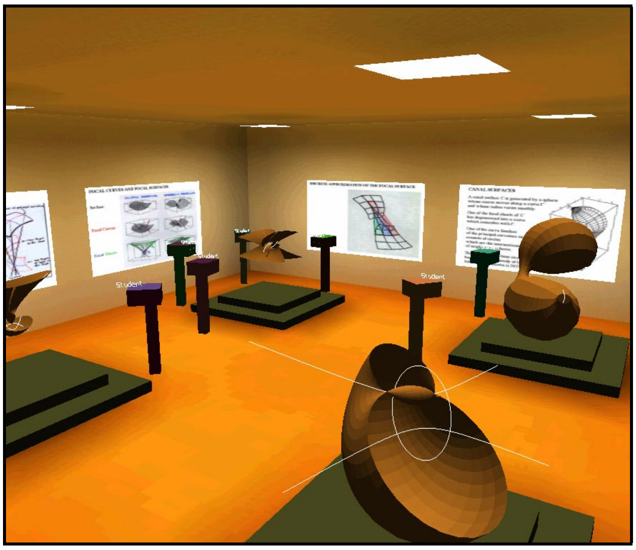

Figure 1. *A CyberMath exhibition on focal surfaces.*

RELATED WORK

Virtual Reality (VR) systems for teaching in schools has been investigated since the early 90s. These include systems for teaching students about physics [10], algebra [3], physiology [19], color science [24] and the greenhouse effect [15], systems for exploring cultural heritage objects [25] and systems for teaching children with disabilities [5]. In [14], a system that allows users to take part in virtual field trips is described. The QuickWorlds program [18] provides schoolteachers with interactive VR models on-demand. Most of these systems are examples of immersive VR systems where the user is in a CAVE environment [8] or wears a head-mounted display.

The rationale behind most VR-based systems for education is the constructivist learning theory, which assumes that students construct their own understanding of what they are studying and that knowledge construction is a collaborative process. Thus, it would seem appropriate to use VR to allow students to discover and experience objects and phenomena in ways that they cannot in do real life.

There is convincing evidence that students can learn from both immersive and non-immersive VR [26]. However, there are also results that indicate that fully immersive VR systems are more effective for learning than their non-immersive counterparts [7]. On the other hand, immersive VR systems are expensive, fragile, hard to integrate in school environments and can induce motion sickness [11]. It is unclear whether the advantages of desktop VR can outweigh the benefits of fully immersive systems. CyberMath can be used in both conditions, which makes it suitable for further studies of these issues.

From the constructivist learning theory, it also follows that teacher guidance and collaboration between students in these three-dimensional virtual environments is likely to have a positive influence on the learning process [26][16][9]. Several preliminary attempts to verify this claim have been made, both for student pairs [15][20][4] and for larger groups of users [17]. Only limitations in network bandwidth places an upper limit on the number of simultaneous users in CyberMath, which makes it suitable for performing additional studies of collaborative situations.

SYSTEM DESCRIPTION

CyberMath is a shared virtual environment that is built on top of DIVE [6]. DIVE has the ability to display shared interactive three-dimensional graphics as well as to distribute live audio. DIVE can run on a standard desktop PC with an OpenGL-compliant graphics accelerator under Windows NT/2000, equipment that is increasingly common in schools today. DIVE also supports a number of other hardware configurations, ranging from head-mounted displays to CAVE environments. It is possible to allow different users to access the same virtual environment from workstations with different hardware configurations.

In order to encourage exploration and informal visits to the virtual environment, we have chosen to build CyberMath as an exploratorium that contains a number of exhibition areas. These areas are accessed from a central hub that is designed as a lecture hall (figure 2). The lecture hall can be used to show standard PowerPoint presentations and also serves as a point of entrance into the exhibitions and as a meeting place for students and teachers. The lighting in CyberMath was generated with the radiosity method [13], which gives a realistic diffuse look to the environment. Our hope is that this will increase the feeling of being immersed in the virtual environment and also increase the general ”wow”-factor of the application.

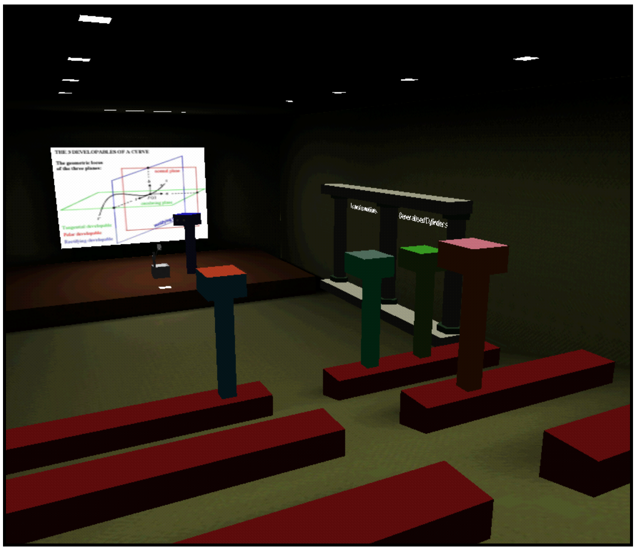

Figure 2. *The CyberMath lecture hall.*

DIVE supports rapid prototyping through Tcl/Tk scripts [23]. We have complemented this support with a Mathematica-to-DIVE conversion utility that can be used to convert standard three-dimensional Mathematica objects and animations to the DIVE file format. It is then straightforward to add Tcl/Tk code to turn the converted Mathematica objects into interactive CyberMath exhibitions. We are planning to develop an exhibition construction tool that will allow teachers without previous Tcl/Tk knowledge to create their own exhibitions.

In CyberMath, the standard DIVE avatars are used to represent visitors in the virtual environment. These are named user embodiments that have been designed with the intention to make it easy for users to recognize their position and orientation [1]. However, DIVE places no restriction on the geometry and appearance of user embodiments, so it is straightforward to individualize the avatars further.

When a user points to an object in the environment using the computer mouse, the avatar will indicate this through a ”laser pointer” – a red line from the eye of the avatar through the indicated point on the object. Each avatar also has a sound indicator that is activated when its corresponding user speaks into the computer microphone. Objects in the exhibition areas can be rotated and translated by using the computer mouse. There are action buttons situated next to some objects that when clicked on, can control animations and toggle the wireframe representation of the objects.

It is also possible to associate URLs with CyberMath exhibition objects. When a user clicks on such an object, its URL is opened in a WWW browser. This makes it easy to offer additional information on the exhibition objects (such as mathematical formulae and links to other relevant WWW pages).

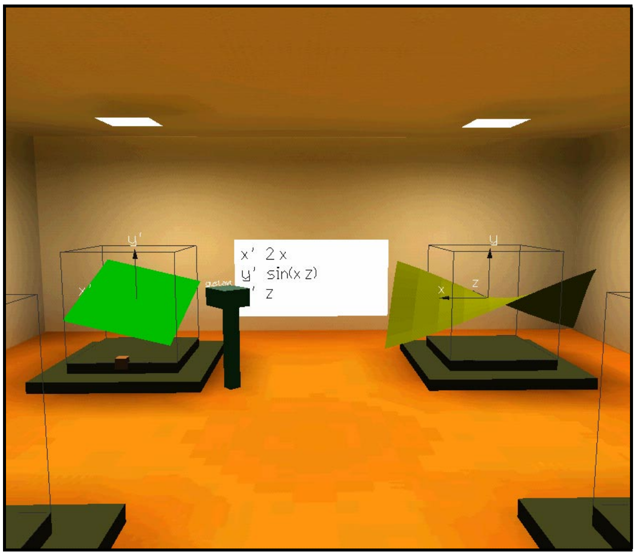

Figure 3. *The interactive transformations exhibition. The user is manipulating the green plane in the domain on the left and the corresponding transformed surface appears in yellow on the right. The transformation is printed on the wall between the two coordinate systems.*

At the time of writing, four example exhibition areas in the exploratorium have been completed. Their respective content is:

Interactive transformations. An $\mathbf{R}^3 \rightarrow \mathbf{R}^3$ transformation maps a three-dimensional domain point to another three-dimensional range point. In this exhibit, users can investigate the effects of any such transformation on different mathematical entities, including points, lines, planes, boxes and spheres. The user can manipulate the entities in the domain and immediately see the results of the transformation, either in a separate coordinate frame or in the same coordinate frame as the untransformed surface (figure 3). This makes it possible to explore transformations in a new way and get an intuitive sense for how a specific transformation works. We believe that this increases the cognitive contact with the mathematical ideas behind the transformation formulae.

Generalized cylinders. In this area, users can learn how to generalize the lathing process and construct complex three-dimensional surfaces using methods from differential geometry [22]. The exhibition includes a number of three-dimensional animations and wall posters that illustrates these methods (figure 4). Differential geometry is usually taught at the post-graduate level (if at all), but we have found that it is relatively easy to use CyberMath to explain how these differential geometric surfaces are created and illustrate some of their applications to undergraduate students (see below).

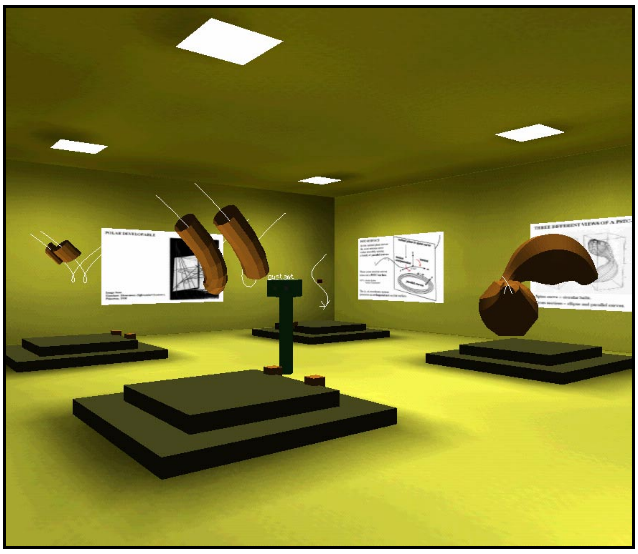

Figure 4. *The exhibition on generalized cylinders.*

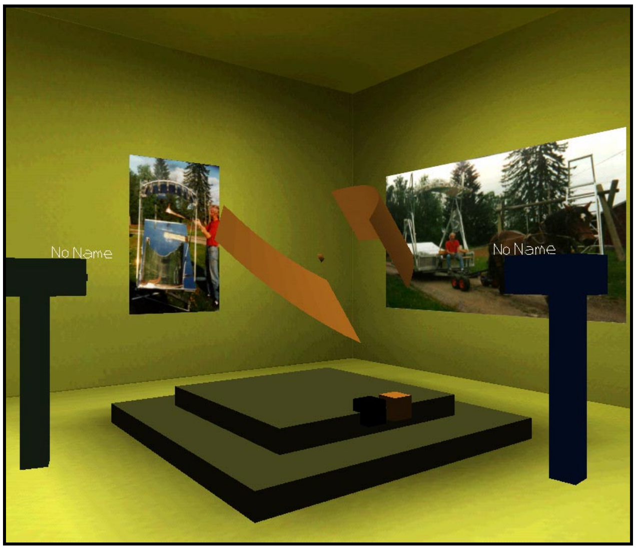

Figure 5. *The cylindrical optics exhibition.*

Cylindrical Optics. In this exhibit we show an application of differential geometry to optics, which is of great practical importance e.g. in the field of solar energy. Two cylindrical mirrors can be arranged in space so as to perform the same function as a parabolic disc, i.e. to focus a planar wavefront into a point. This is done in two steps, using two flat sheets that can be bent into the right shape (figure 5). This double-cylindrical point focus configuration is convenient because it is easier (and thus cheaper) to manufacture than a corresponding parabolic disc. Another important advantage is that the focal point can be placed outside of the spatial region between the mirrors, where it is freely available to do work [21].

Focal Surfaces. In this exhibition we illustrate another area of differential geometry which is concerned with characterizing the geometric structure of the normals to a given surface. In general, these normals are tangent to two other surfaces, called the focal sheets. Taken together, the focal sheets form the so-called focal surface of the given surface [21]. The exhibition shows examples of the general configuration, as well as the three kinds of degenerations that can occur (figure 1).

USABILITY TESTING AND RESULTS

We have completed two initial usability tests, one small test at our lab with two students and one larger test with thirteen students. In both tests, a mathematics teacher from the Royal Institute of Technology (that is familiar with CyberMath) guided the students through the generalized cylinders exhibition hall. The teacher was in a separate physical location and all students were sitting at different workstations in one room. The students were between 22 and 27 years old and were undergraduates in computer science at the University of Uppsala and at the Royal Institute of Technology in Stockholm. The participants in the first test were interviewed and the participants in the second test answered a questionnaire.

The results suggest that the guided tour was successful and that it is easy to get a sense for complex shapes in CyberMath. A majority of the users felt that they were aware of other users in the virtual environment and that the environment was inviting.

Some users reported problems with navigation and that the avatars of other users hid their view. We are currently investigating different ways of improving the navigation in CyberMath. Also, we will add an option to make avatars transparent in an attempt to solve the visibility issue.

Several users specified that they felt immersed in the environment, even though no special immersive VR hardware was used. One explanation could be the abundance of three-dimensional computer games and visualization tools that are available for standard consumer-level computers today. Users with experience of such systems may find it easier to achieve the necessary suspension of disbelief than users who have no previous experience of virtual environments.

It is clear that CyberMath is exciting and motivating for both teachers and students. A large majority of the test participants thought that CyberMath would be suitable for everyday teaching. However, for these initial tests we have not attempted to assess to what extent the students have a retained long-term knowledge of the exhibition content.

FUTURE WORK

At the moment, it is unclear if the positive response we received in our initial tests is due to the novelty of the technology. We also do not know if participants in CyberMath gain long-term knowledge from their explorations. In order to find an answer to these questions, we are planning a large deployment of CyberMath at the Royal Institute of Technology and a series of new usability tests, where traditional lecturing will be compared to exploration-based teaching in CyberMath. Another possible future area for research is to examine the tradeoffs between immersive VR systems (head-mounted displays, CAVE) and less expensive display alternatives (desktop monitors, wall-projections). We intend to build more exhibitions, including one that presents elementary mathematics and one that introduces geometric algebra. We will use results from research on awareness and accommodation in virtual environments to further guide the design of these exhibition areas. We would also like to study to what extent visual realism influences the learning process.

SUMMARY

We have described CyberMath, a shared virtual environment for mathematics exploration. We believe that our system has the potential to help motivate students of mathematics and to make it easier for teachers to explain diffcult mathematical concepts. The system is built to support the teaching of many mathematical subjects, ranging from elementary school levels to post-graduate levels and supports a variety of teaching styles. The results of two initial usability tests suggest that it is possible to successfully use CyberMath to explain advanced mathematical concepts.

REFERENCES

1. Benford, S., Bowers, J., Fahlén, L. E., Greenhalgh, C. Snowdon, D. User Embodiment in Collaborative Virtual Environments. In *Proceedings of CHI '95*, 242-249.
2. Bergsten, C. (ed.) *Datorstödd eller datorstörd matematikundervisning*. The Swedish National Agency for Higher Education, Report No. 1999:4S, 1999 (in Swedish).
3. Bricken, W. Spatial Representation of Elementary Algebra. In *Proceedings of the 1992 IEEE Workshop on Visual Languages*, 55-62.
4. Brna, P., Aspin, R. Collaboration in a Virtual World: Support for Conceptual Learning? In *Proceedings of the IFIP WG 3.3 Working Conference "Human-Computer Interaction and Educational Tools"*, 113-123.
5. Brown, D. J., Kerr, S., Wilson, J. R. Virtual Environments in Special-Needs Education. *Communications of the ACM*, 40 (8), August 1997, 72-75.
6. Carlsson, C., Hagsand, O. DIVE - A Multi User Virtual Reality System, In *Proceedings of IEEE VRAIS '93*, 394-400.
7. Cronin, P. *Report on the Applications of Virtual Reality Technology to Education*. HRHC, University of Edinburgh, February 1997. http://www.cogsci.ed.ac.uk/~paulus/vr.html
8. Cruz-Neira, C., Sandin, D. J., DeFanti T. A. Surround-Screen Projection-Based Virtual Reality: The Design and Implementation of the CAVE. In *Proceedings of SIGGRAPH '93*, 135-142.
9. Dede, C. The Evolution of Constructivist Learning Environments: Immersion in Distributed, Virtual Worlds. In *Educational Technology*, 35 (5), 1995, 46-52.
10. Dede, C., Salzman, M. C., Loftin, R. B. ScienceSpace: Virtual Realities for Learning Complex and Abstract Scientific Concepts. In *Proceedings of IEEE VRAIS '96*, 246-252.
11. Dede, C., Salzman, M., Loftin, R. B., Ash, K. Using Virtual Reality Technology to Convey Abstract Scientific Concepts. In Jacobson, M. J., Kozma, R. B. (Ed.), *Learning the Sciences of the 21st Century: Research, Design, and Implementing Advanced Technology Learning Environments*. Lawrence Erlbaum, 1997.
12. Gonzales, P., Calsyn, C., Jocelyn, L., Mak, K., Kastberg, D., Arafeh, S., Williams, T., Tsen, W. *Pursuing Excellence: Comparisons of International Eight-Grade Mathematics and Science Achievement from a U.S. Perspective, 1995 and 1999*. U. S. Department of Education, National Center for Education Statistics, Report No. NCES 2001-028, December 2000.
13. Hanrahan, P., Salzman, D., Aupperle, L. A Rapid Hierarchical Radiosity Algorithm. In *Proceedings of SIGGRAPH '91*, 197-206.
14. Harada, Y., Nosu, K., Okude, N. Interactive and Collaborative Learning Environment using 3D Virtual Reality Content, Multi-Screen Display and PCs. In *Proceedings of the IEEE 8th International Workshops on Enabling Technologies: Infrastructure for Collaborative Enterprises, 1999 (WET ICE '99)*, 238-244.
15. Jackson, R. L. Peer Collaboration and Virtual Environments: A Preliminary Investigation of Multi-Participant Virtual Reality Applied in Science Education. In *Proceedings of the ACM 1999 Symposium on Applied Computing*, 121-125.
16. Jin, Q., Yano, Y. Design Issues and Experiences from Having Lessons in Text-Based Social Virtual Reality Environments. In *Proceedings of the 1997 IEEE International Conference on Computational Cybernetics and Simulation*, vol. 2, 1418-1423.
17. Johnson, A., Roussos, M., Leigh, J., Vasilakis, C., Barnes, C., Moher, T. The NICE Project: Learning Together in a Virtual World. In *Proceedings of IEEE VRAIS '98*, 176-183.
18. Johnson, A., Moher, T., Leigh, J., Lin, Y-J. QuickWorlds: Teacher-Driven VR Worlds in an Elementary School Curriculum. In *Proceedings of SIGGRAPH '00 Educators Program*, 60-63.
19. Kohsaka, Y., Hashimoto, K., Shibata, Y., Katsumoto, M. Flexible Multimedia Lecture Supporting System based on Extended Virtual

Reality Space. In *Proceedings of the IEEE 1999 International Workshops on Parallel Processing*, 614-619.

20. Moher, T., Johnson, A., Ohlsson, S., Gillingham, M. Bridging Strategies for VR-Based Learning. In *Proceedings of CHI '99*, 536-543.

21. Naeve, A. *Focal Shape Geometry of Surfaces in Euclidean Space*. Dissertation No. CVAP-130 / TRITA-NA-P9319, Department of Numerical Analysis and Computing Science, The Royal Institute of Technology, Stockholm, 1993.

22. Naeve, A., Eklundh, J. O. Representing Generalized Cylinders. In *Proceedings of the 1995 Europe China Workshop on Geometric Modeling and Invariants for Computer Vision*, Xi'an, China, April 1995, pp.63-70. (http://kmr.nada.kth.se/papers/MathematicsEducation/RepresentingGenCyls.pdf)

23. Ousterhout, J. K. Scripting: Higher Level Programming for the 21st Century. *IEEE Computer*, March 1998.

24. Stone, P. A., Meier, B. J., Miller, T. S., Simpson, R. M. Interaction in an IVR Museum of Color. In *Proceedings of SIGGRAPH '00 Educators Program*, 42-44.

25. Terashima, N. Experiment of Virtual Space Distance Education System Using the Objects of Cultural Heritage. In *Proceedings of the 1999 IEEE International Conference on Multimedia Computing and Systems*, vol. 2, 153-157.

26. Winn, W. *The Impact of Three-Dimensional Immersive Virtual Environments on Modern Pedagogy*. University of Washington, HITL, Report No. R-97-15, 1997.